SOLACTIVE
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MINIMUM DOWNSIDE VOLATILITY INDICES

White Paper

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1 INTRODUCTION

"*Analyses based on S1 tend to produce better portfolios than those based on V. Variance considers extremely high and extremely low returns equally undesirable. An analysis based on V seeks to eliminate both extremes. An analysis based on SE, on the other hand, concentrates on reducing losses.*"(Markowitz, 1959, p. 194)

To minimize a portfolio's volatility, one usually optimizes the variance-covariance matrix of the stock returns in question. Doing so, one considers both negative and positive deviations from the mean returns equally. However, investors are interested in minimizing negative returns. A more appropriate risk measure, therefore, should only consider returns that fall below a certain threshold. In 1959, Markowitz already suggests the semi-variance as a smart alternative to the variance. The square-root of the semi-variance, called downside volatility, measures the volatility of returns below that threshold.

Consider, for example, two portfolios realizing the following sets of returns: A = [-0.1| -0.1| -0.1] and B = [0.02 | 0.1 | 0.03]. The respective volatilities are 0 and 0.04, i.e. portfolio A is considered the less risky investment by the classic standard deviation framework. The downside volatility, on the other hand, suggests that portfolio B is the less risky investment, as the resulting downside volatilities are 0.1 and 0, respectively. This result is more in line with what a typical investor would prefer.

In other words, the downside volatility produces more consistent risk figures. To minimize a portfolio's risk in terms of downside volatility, we calculate the semi-covariance matrix of asset returns as introduced by Estrada (2008). Using this heuristic definition, we can optimize the semi-covariance matrix and find a closed form solution that minimizes the downside volatility of the portfolio. As a result, we obtain an index that has minimum risk, defined in a more intuitive way. The remainder of this paper focuses on the application of the MDV strategy on US large caps and is organized as follows. In section 2 the theory behind the optimization is introduced. Section 3 contains analytics on the Solactive US Large Cap and the Solactive US Large Minimum Downside Volatility Index. Section 4 concludes.

2 THEORY

To find the weights that minimize the portfolio's downside volatility we solve the following optimization problem:

$$\min(\omega' \cdot \Sigma \cdot \omega)$$

Equation (1)

where w is a vector of weights. The semi-covariance matrix $\sum$ is defined as,

$$\sum ijB = \frac{1}{T} \cdot \sum_{t=1}^{T} [\,\mathrm{Min}(R_{i,t} - B, 0) \cdot \mathrm{Min}(R_{j,t} - B, 0)\,]$$

Equation (2)

where T is the number of observations, $\mathrm{R}_{\mathrm{i,t}}$ is the return of asset i at time t, and B is the threshold return. For our indices, we set B equal to zero, i.e. we are minimizing risk defined as volatility of negative returns. The optimization is solved subject to a set of constraints. First, the sum of weights of all index members must be equal to one.

$$\sum_{i=1}^{n} \omega_i = 1$$

Equation (3)

Secondly, we fix the number of final index members, N. This is implemented by *Equations (4)* and *(5)*.

$$\sum_{i=1}^{n} y_i = \mathrm{N}$$

Equation (4)

$$y_i \in \{0,1\} \; i = 1, \dots, n$$

Equation (5)

The latter assigns each stock either a value of one when it is included in the index or a zero otherwise. The former makes sure that the sum over these Boolean values equals the specified number of index components. Further, we introduce an upper and lower bound for the individual stock weight.

$$\omega_i^{min} \leq \omega_i \leq \omega_i^{max} , i = 1, \dots, n$$

Equation (6)

Equation (7) limits the weight that can be invested in a certain sector relative to the sector allocation of the benchmark by setting individual upper and lower sector bounds, s_j.

$$s_j^{min} \leq \sum_{i \in s(j)}^{n} w_i \leq s_j^{max}$$

Equation (7)

Where s_j denotes the set of stocks that are included in sector j. A similar requirement can be implemented for the country allocation:

$$c_k^{min} \leq \sum_{i \in C(k)}^{n} w_i \leq C_k^{max}$$

Equation (8)

where c_k are all stocks that are part of country k. The maximum one-way turnover (OWT) constraint is implemented as in *Equation (9)*:

$$\frac{1}{2} \cdot \sum_{i=1}^{n} | w_{i,t} - w_{i,t-1} | \leq OWT$$

Equation (9)

A problem often encountered when estimating the (semi-)covariance matrix is the curse of dimensionality, i.e. we typically have a large number of stocks available but comparatively few observations. As a result, the semi-covariance matrix would be estimated with large estimation errors. This would deteriorate the out-of-sample performance of our resulting portfolio. Common remedies to this problem are the usage of factor models or shrinkage (compare Ledoit and Wolf, 2004) [Ref. 5] to estimate the (semi-)covariance. However, as shown e.g. by Jagannathan and Ma (2003) [Ref. 4] or Frost and Savarino (1988) [Ref. 2], imposing a no-shortsales constraint into the optimization problem leads to portfolios that perform as well as portfolios that use factor models or shrinkage when estimating the (semi-)covariance. In fact, they also show that introducing a no-shortsales constraint when using factor models or shrinkage for estimation even hurts the out-of-sample performance of the resulting portfolios. Therefore, as we do not allow short-selling in our index we refrain from using factor models or shrinkage when estimating the semi-covariance matrix.

To solve the above optimization, we use the Outer-Approximation Algorithm as described in Hemmecke et al. (2010) [Ref. 3]. In the first step we solve the quadratic programming problem of the form

$$\frac{min}{x} \frac{1}{2} \cdot \; x^T H x + f^T x \text{ such that} \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

The second step solves the linear programming problem,

$$\frac{min}{x} f^T x \text{ such that} \begin{cases} A \cdot x \leq b, \\ Aeq \cdot x = beq, \\ lb \leq x \leq ub. \end{cases}$$

Both problems are solved using interior-point algorithms.

3 INDEX ANALYTICS

This section presents results of a historical simulation (backtest) of the Solactive US Large Cap Minimum Downside Volatility Index (SOL US LC MDV) starting in February 2004. We compare it to the starting universe, which is represented by the float market cap weighted Solactive US Large Cap. *Figure (1)* displays the results. *Table (1)* displays the detailed statistics.

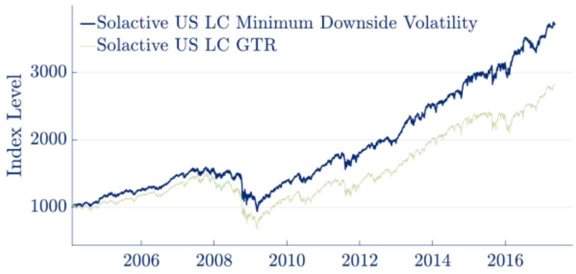

Figure (1): Backtest Results

	SOL US LC MDV	SOL US LC
Mean return	10.22%	7.99%
Std. Dev.	14.58%	18.68%
Downside Dev.	10.22%	13.29%
Max. Drawdown	-41.42%	-54.73%
Sharpe Ratio	0.70	0.43
Sortino Ratio	1.00	0.60

Table (1): Backtest Results (annualized)

The following parameters have been used for calculation of the backtest of the SOL US LC MDV.

Starting Universe: Solactive US Large Cap Index

Index Currency: USD

Index Type: Gross Total Return

Minimum Stock Weight: 0.15%

Maximum Stock Weight: 3.00%

Number of Stocks: 100

Minimum 6-month ADV: $ 10 million

Relative Sector Capping: ± 2.50 percentage points (relative to starting universe)

Maximum One-Way Turnover: 10.00 percentage points

The first thing to notice is that the downside volatility of our approach is substantially lower than the one of the SOL US LC. This reflects the success of our optimization routine. As a consequence, the risk-adjusted returns, as illustrated by the Sharpe- and Sortino-Ratio, are distinctly higher. Further, our realized maximum drawdown is significantly reduced, which is also a result of the risk minimization. The ratio of the SOL US LC MDV against the SOL US LC approach, shown in *Figure (2)*, increases especially in turmoil periods. Note, for instance, how the ratio starts to rise in 2007 when the subprime mortgage crisis began.

Figure (2): Ratio SOL US LC MDV Index over SOL US LC Index

Moreover, *Figure (3)* shows that the strategy does not only work in extreme market conditions, but also during more modest bear markets.

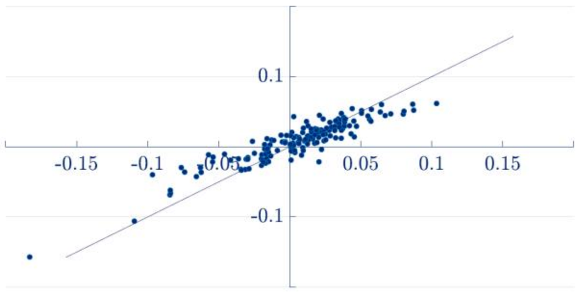

Figure (3): Scatterplot of SOL US LC MDV against SOL US LC

This becomes obvious as most of the negative returns are located above the 45° line indicating a higher return of the SOL US LC MDV compared to the SOL US LC.

The sector allocation shown in *Figure (4)* exhibits that as of the most recent selection the largest parts are invested in Financials, Information Technology and Consumer Staples. During the backtesting period sectors like Utilities, Consumer Staples or Real Estate were typically among the most prominent ones.

This is shown in *Figure (5)* which illustrates the difference in the sector allocation of the SOL US LC MDV in comparison to the SOL US LC. It can be observed that the strategy avoids excessive sector tilts and tracks the allocation of its benchmark closely.

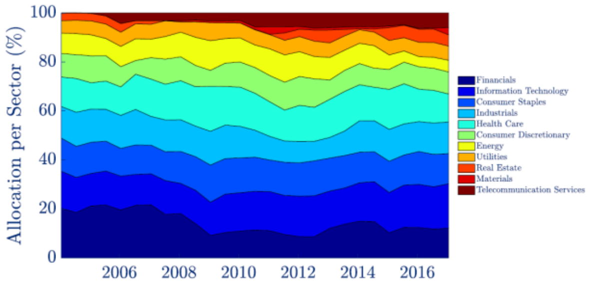

Figure 4: Historic Sector Allocation (%)

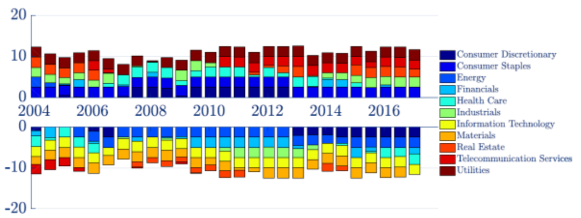

Figure (5): Relative Sector Allocation of the SOL US LC MDV against the SOL US LC (%))

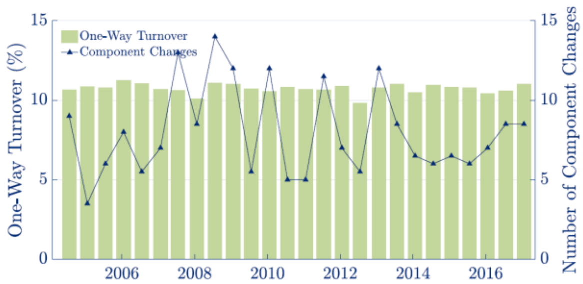

Figure 6: Historic Turnover of the SOL US LC MDV (%)

Figure (6) furthermore shows that the turnover constraint has been achieved at every selection day, leaving the historical one-way turnover at roughly 20% per annum.

4 CONCLUSION

We introduce a new approach of risk minimization in the index context. While standard deviation, as used in classic portfolio theory, punishes positive and negative deviations from mean returns equally, the downside volatility only considers negative returns when calculating an asset's risk. By optimizing our starting universe according to downside volatility, we manage to create a new index that has minimum risk and superior performance figures. In other words, the Solactive US Large Cap Minimum Downside Volatility Index generates lower downside volatilities, lower maximum drawdowns, and higher risk-adjusted returns compared to classical volatility optimized indices.

5 REFERENCES

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Ref. 3 Hemmecke, R., Koeppe, M., Lee, J., & Weismantel, R., (2010). Nonlinear integer programming. 50 Years of Integer Programming 1959-2008. Springer Berlin Heidelberg.

Ref. 4 Jagannathan, R., & Tongshu M., (2003). Risk reduction in large portfolios: Why imposing the wrong constraints helps. The Journal of Finance, 58.4.

Ref. 5 [Ledoit, Olivier, & Michael Wolf, (2004). A well-conditioned estimator for large-dimensional covariance matrices. Journal of Multivariate Analysis, 88.2.

Ref. 6 [6] Markowitz, H., (1968). Portfolio selection: Efficient diversification of investments. Yale University Press, 16.

All numbers are calculated by Solactive as of Q4 2017.

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