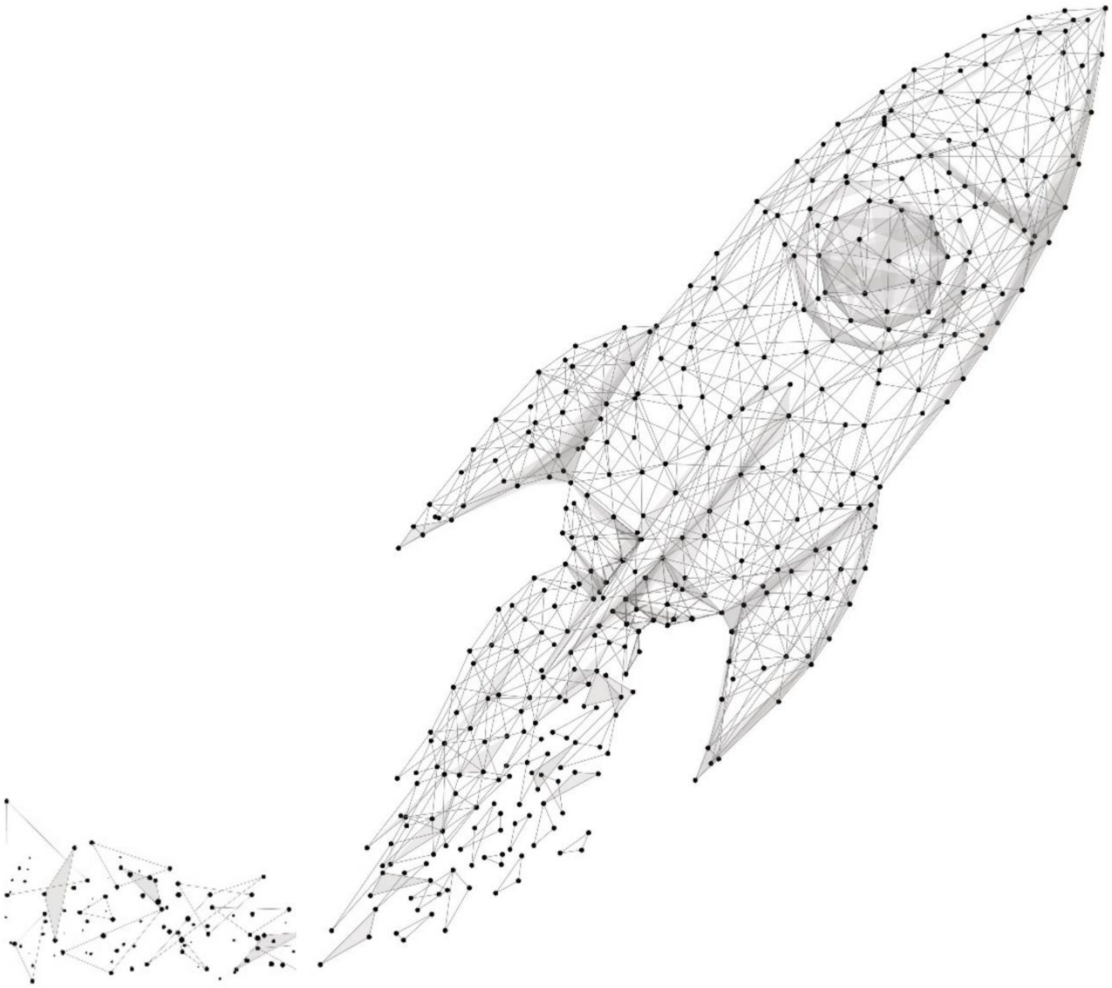
(High Dividend)

Maximum Upside Volatility Indices

Financial Index Engineering for Structured Products



White Paper

April 2018

Introduction

This report provides a detailed and technical look "under the hood" of an index family that we have designed specifically with structured products in mind: the **Solactive High Dividend Maximum Upside Volatility Indices.** The starting objective is to create a portfolio of stocks exhibiting very high upside volatility and yielding a consistently high dividend yield. This index family would be particularly relevant for structured products payoffs with a short volatility profile, such as autocallables or discount certificates among others.

This paper starts by providing the theoretical background and then describes the optimization process in detail. Finally, it shows how the methodology is applied step-by-step on one index, which is live and fully investible: the **Solactive High Dividend Upside Volatility Euro 5% Adjusted Return Index** (Bloomberg Ticker: SOLHDMUV Index).

The upside of volatility

"Analyses based on semi-variance tend to produce better portfolios than those based on variance. Variance considers extremely high and extremely low returns equally undesirable." (Markowitz, 1959, p. 194).

To minimize or maximize portfolio volatility, the common procedure is to optimize the variance-covariance matrix (also referred to as variance matrix) of the portfolio (more details in the next section). In this case, both negative and positive deviations from the mean return would be treated equally.

Consider two stocks realizing the following sets of returns: A = [-10% -15% -20%] and B = [2%, 3%, 4%]. The volatilities of A and B are 5% and 1% respectively. If we simply want the most volatile stock, we would choose A, even though all of its returns are negative.

Enter, Upside Volatility. To calculate upside volatility, the first step is to disregard all negative returns (i.e. to replace all negative returns with zeroes). In this case, the upside volatility of A and B is 0% and 1% respectively. If we want the most volatile stock while differentiating between desirable and undesirable volatility, we would now choose B.

Variance matrix vs. semi-variance matrix

Given two assets i and j, the traditional **variance matrix** is defined as:

$$\sum ij = \frac{1}{T-1} \cdot \sum_{t=1}^{T} [\, (R_{it} - \mu_i) \cdot (R_{jt} - \mu_j) \,] \qquad \textit{Equation (1)}$$

where T is the number of observations, R_{it} is the return of asset i at time t, and μ_i is the mean return of asset i over the period of observation.

For our index, we rely on the definition of **semi-variance** (also known as semi-covariance) that Estrada introduced in 2008 [Ref.2]. Estrada calculates semi-variance in the same way as the equation above, albeit with two differences:

- Only positive returns are taken into account for upside volatility (or negative returns for downside volatility).
- We care about deviations away from 0% (i.e. the threshold between positive and negative returns), not about deviations from the mean returns (denoted as μ_i and μ_j in the formula above, which will also be set to 0%).

Therefore, given two assets i and j, the **semi-variance matrix** is defined as:

$$\sum ij = \frac{1}{T-1} \cdot \sum_{t=1}^{T} [\, max(R_{it}\,, 0) \cdot max\ \ (R_{jt}\,, 0)\,] \qquad \text{Equation (2)}$$

where T is the number of observations, and $max\ (R_{it}\ ,0)$ is the maximum between zero and the return of asset i at time t (equivalent to replacing negative returns with zeroes). As a side note, to calculate for downside volatility, $max\ (R_{it}\ ,0)$ would simply be replaced by $min\ (R_{it}\ ,0)$.

The optimization process

To find the weights that maximize the portfolio's upside volatility, we solve the following optimization problem[1]:

$$max\ \ (\ \omega' \cdot \Sigma \cdot\ \omega\ ) \qquad \text{Equation (3)}$$

where ω is the vector of weights outputted by the optimizer, ω' is the same vector but transposed, and the semi-variance matrix Σ is defined above in *Equation (2)*.

The optimization is solved subject to a set of constraints.

1. First, the sum of the weights of all index members must be equal to one:

$$\sum_{i=1}^{N} \omega_i = 1 \qquad \text{Equation (4)}$$

2. We introduce a minimum weight and a maximum weight for all individual stocks:

$$\omega_i^{min} \leq\ \omega_i\ \leq\ \omega_i^{max}\,, \qquad i = 1, \dots, n \qquad \text{Equation (5)}$$

3. A very important starting objective of this index family is to demonstrate consistently high dividend yields. Therefore, we introduce another constraint – a portfolio level dividend yield floor at every selection:

$$\sum_{i=1}^{N} \omega_i \cdot div_i \geq Div^{min} \qquad \text{Equation (6)}$$

   where div_i is the trailing dividend yield for stock i, and Div^{min} is the portfolio-level dividend yield floor.

4. Finally, we add a sector constraint, as this index should be fully investible and should avoid a high concentration into one sector.

$$\sum_{i=1}^{n_j} \omega_i \leq SW_j^{max} \qquad \text{Equation (7)}$$

   where n_j is the number of stocks in sector j, and SW_j^{max} is the maximum weight allocation to sector j. This constraint is set to all the sectors.

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[1] We solve the optimization problem using the interior-point algorithm (*fmincon* in Matlab).

Methodology walk-through: SOLHDMUV Index

In this section, we use SOLHDMUV Index (Solactive High Div Upside Vol Euro 5% Adj. Return Index) as an example for the application of this optimization process.

The idea is simple: start with a stock pool of liquid, high dividend paying stocks. Run the optimizer on the entire stock pool to obtain the optimal weights. Kick out the stocks that have the lowest weight (i.e. the stocks that contribute the least to portfolio volatility). Run the optimizer again, kick out some stocks, and keep reiterating this process until we arrive at the desired number of index members (50 stocks in this case).

While some parameters such as liquidity filters, number of components, weight caps, or dividend treatment (i.e. Price Return, Adjusted Return, etc.) could differ for different indices in this family, the process itself remains the same, as detailed in the steps below.

1. **Select Starting Universe:** the Eurozone investible universe for this version of the index.
2. **Apply ADV Filter:** minimum 6-month ADV of EUR 20 mil. Less liquid stocks are excluded.
3. **Rank according do dividend yield:** select top 75 stocks in terms of dividend yield. The rest are excluded.

The idea now is to find the combination of 50 stocks (and their weights) out of these 75, which together exhibit optimal upside volatility – the highest achievable while respecting constraints such as dividend yield floors, sector exposure caps, and individual min/max weight caps.

4. **Start the optimization process** using these 75 stocks:
   A. Calculate daily returns looking back 252 business days
   B. Replace negative returns with zeros (since we are interested in upside deviation). This return matrix is denoted R (whose dimensions in this example are 252 by 75).
   C. As described above in *Equation (2)*, calculate semi-variance matrix as $\Sigma = \frac{R' \cdot R}{T-1}$, where T is the number of observations (252 business days in this case), and R' is the return matrix transposed.
   D. The weights vector that the optimizer returns is denoted ω. The optimizer would then solve $max\ \omega' \cdot \Sigma \cdot\ \omega$ (i.e. maximizing portfolio volatility), with the following constraints:
      i. Sum of weights adds up to 1.
      ii. Min/Max weights of 0% and 5% for individual stocks.
      iii. Dividend yield floor at portfolio level of 5%.
   E. After the optimizer finds the weights for these 75 stocks, we drop the 5 stocks with the lowest weights. We then repeat steps C and D **until 50 stocks remain** (optimize, drop 5, optimize, drop 5, etc).
   F. When 50 stocks are left, we reiterate the optimization process described above from steps C to D, with two alterations made to step D:
      ii. Min/Max weights of 0.25% and 5%.
      iv. sector constraint: absolute cap of 25%.

In case the optimizer does not find a solution that satisfies all the constraints, the algorithm relaxes the dividend yield constraint by 0.25% and the sector constraint by 5% and then it tries again. It will relax these constraints as many times as needed until a solution is found (over the back-tested period, constraint relaxation didn't occur).

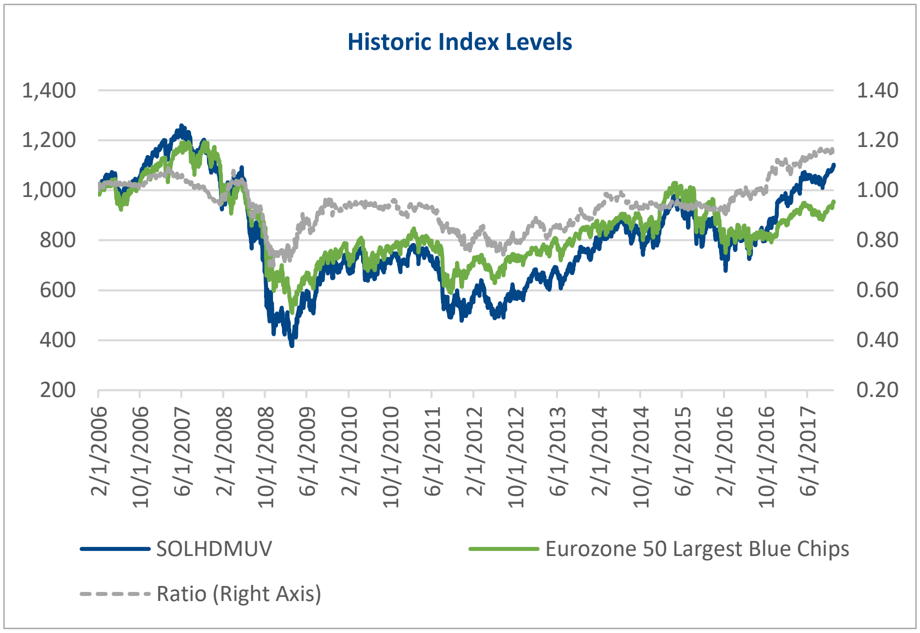
As an additional note, we assigned much more importance to the dividend yield than to the sector constraint. In other words, we don’t mind the sector exposure to increase slightly **if doing so ensures a high dividend yield and upside volatility**. For example, we do not want the dividend yield or the volatility to drop by 1% just to cap sectors at 25% and not 30%.

In this light, we introduced the sector constraint in the final step (F) and not in the stock-dropping-steps (C and D) since the main objective is to eliminate stocks based on their (low) contribution to portfolio (upside) volatility and dividend yield. The optimizer will only focus on the dividend yield and on the volatility, and it will “worry” about the sector exposure only after the 50 final index members have been selected. The algorithm will not kick out stocks just to satisfy the sector constraint.

Did it work? SOLHDMUV Index

We now take a look at the performance of Solactive High Div Upside Vol Euro 5% AR Index, which is a direct output of the algorithm described above.

As aforementioned, the objective of this index is to achieve a pickup in volatility and high dividend yields. Additionally, the index should be fully investible through high liquidity requirements and weight caps. In order to judge its performance, we compare our index to a portfolio made of the 50 largest blue-chip stocks in the Eurozone.

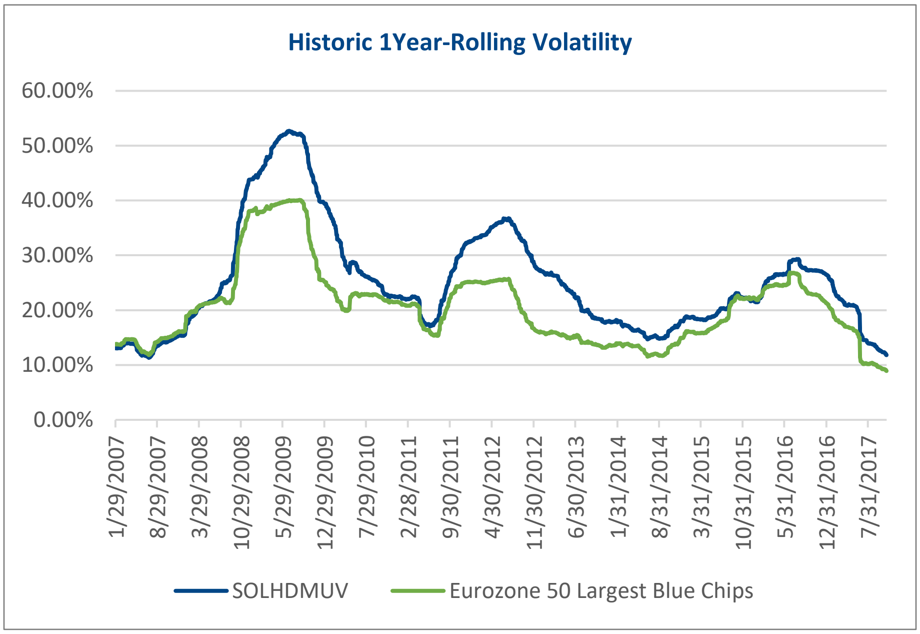


The adjacent graph shows that the index levels are in line with its benchmark. It actually slightly outperforms **despite the 5% synthetic drag** in this adjusted return version (simple Price Return version available as well).

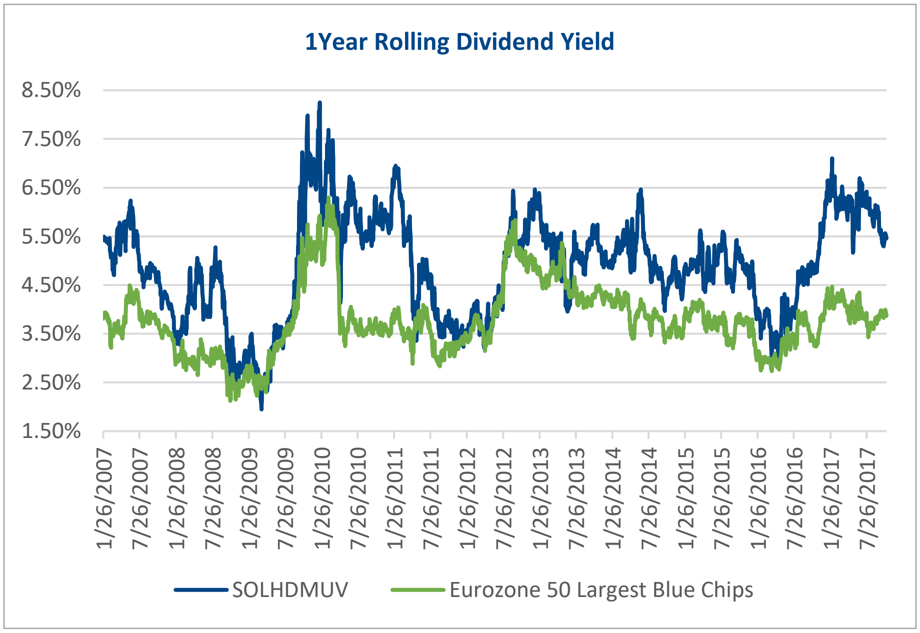
While it exhibits larger drawdowns during bear markets, it recovers faster during up-markets, eventually recovering and overtaking the benchmark. This is visually represented by the outperformance ratio in the chart (right axis).

Despite the performance drag and the high volatility, our index still generates sufficient outperformance to realize the better Sharpe ratio.

	SOLHDMUV (5% AR)	**Eurozone 50 Largest Blue Chips (PR)**
Mean	0.77%	-0.40%
Standard Deviation	26.21%	21.17%
Downside Deviation	18.64%	15.10%
Max Drawdown	-70.09%	-57.35%
Sharpe Ratio	0.03	-0.02
Sortino Ratio	0.04	-0.03



As expected, our index manages to display a consistent pickup in the one year rolling volatility, plotted on the left.



Finally, our index achieves a significantly higher dividend yield relative to the Eurozone benchmark.

Please note that this graph shows realized dividend yields. The dividend yield constraint of 5% refers to 12-month trailing dividend yield. There is no guarantee that a stock will pay the same (high) dividends in the future. Nevertheless, the dividend yield constraint still "does its job" well, as we can observe in the adjacent graph.

References

[Ref.1] Markowitz, H., (1968). *Portfolio selection: Efficient diversification of investments*. Yale University Press, 16.

[Ref.2] Estrada, J., (2008). *Mean-semivariance optimization: A heuristic approach*. Journal of Applied Finance, 18.1.

Contact

SOLACTIVE AG
German Index Engineering
Guiollettstr. 54
60325 Frankfurt am Main
www.solactive.com

Timo Pfeiffer, Head of Research & Business Development
+49 (69) 719 160 320
pfeiffer@solactive.com

Emanuel Cozmanciuc, Quantitative Research Analyst
+49 (69) 719 160 313
cozmanciuc@solactive.com

Fabian Colin, Head of Sales
+49 (69) 719 160 220
colin@solactive.com

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